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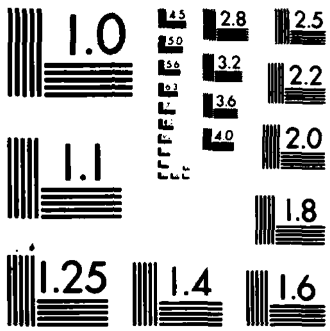

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BIFURCATION AND FEEDBACK IN AIRCRAFT DYNAMICS

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1. Historical Remarks on HOPF Bifurcation in Aircraft Dynamics

Modern qualitative theory of differential equations originated with the work of Poincare, who essentially had arrived at the phenomenon which is presently called Hopf bifurcation. Poincare's work was later developed by Androuov and Pontryagin [1]-[3]] and Hopf [15].

More recently, experience with the aircraft at high angles of attack shows loss of stability for some critical values of parameter (e.g., angle of attack, or the velocity). This phenomenon was interpreted by R. Mehra et al. [25] as the Hopf bifurcation; this interpretation was supported by numerical study for the aircraft H model.

The work of Mehra et al. led naturally to the question as to the effect of control decoupling feedback on the (undesirable) Hopf bifurcation; in particular, does such a feedback eliminate the bifurcation? Among other things, this question will be answered in this report.

2. Historical Remarks on Control Decoupling via Feedback

The problem of control decoupling via feedback together with the closely related problems of disturbance decoupling and invariance has arisen in many engineering applications, and particularly in connection with the aircraft control problem. Since Rozenoer's initial work [25], the subject of control decoupling via feedback has been extensively developed, and a reasonably large body of literature currently exists. Some papers which have been important milestones are Wonham and Morse [33], Tokamaru and Iwai [32], Majumdar and Chaudhury [17], Isidori et al. [16], Hischorn [14], and Byrnes and Krener [7]. In addition to this theoretical work, several applications to problems of aircraft control have been studied including Singh and Schy [27] and work by G. Meyer [20] on the design of an autopilot system for the Bell UH-IH helicopter.

The basic idea behind the theory of control decoupling is quite simple: Suppose there is given a nonlinear control system of the form

$$x = f(x) + u_1 g_1(x) + u_2 g_2(x)$$

$$y_i = h_i(x) \qquad (i=1,2)$$

We wish to consider modifications of the system dynamics using feedback controls $u = \alpha(x) + \beta(x)v$

$$\dot{x} = \tilde{f}(x) + v_1 g_1(x) + v_2 g_2(x) \qquad (2.1)$$

$$y_i = h_i(x) \qquad (i=1,2) \qquad (2.2)$$

where

$$\tilde{f} = f + \sum_{j=1}^{2} \alpha_j \; g_j$$

$$\tilde{g}_i = \sum_{j=1}^{2} \beta_{ji} \; g_j$$

The decoupling problem is to find α and β such that v_i controls y_i and only y_i. (That is, we want v_1 to have no influence on the output y_2 and vice-versa).

Techniques for finding α and β in the case in which f and the h_i's are linear and the g_i's are independent of x are well known and may be found in Wonham [33]. For nonlinear systems, considerably less is known and although the beginnings of the theory date back to 1962, many aspects remain to be understood.

In 1962, L. Rozonoer [25] obtained conditions necessary for invariance (i.e. independence of the output upon the input) in nonlinear systems, by using variational method, similar to the Pontryagin's optimality principle.

H. Tokamaru and Z. Iwai [32], and A. Majumdar and A. Chaudry [17], independently have applied the variational method used by Rozonoer to obtain necessary conditions for non-interacting control, for linear time-variant systems (1968) and for nonlinear systems (1971).

A concrete design for control decoupling feedback for the automatic piloting of a mine hunter boat was given by E. Daclin [8].

All theoretical work mentioned above neglected the existence of constraints on controls in virtually every real-life situation (e.g., a limit on rudder deflections, acceleration rates, etc. The effects of such constraints have been studied by D. Hanson and F. Stengel [12], particularly for systems with two degrees of freedom. Further development of their analysis seems to be of considerable interest.

A different approach to the problem was used by R. Su, G. Meyer and L. Hunt [31], who used nonlinear transformations to reduce the nonlinear problem to the linear one, which can be treated by standard methods, as in Wonham [33]. The main drawback here is that it is not generally possible to carry out this type of linearization; there are both algebraic and topological obstructions which are essentially the same ones that are encountered if one were to try to do nonlinear decoupling directly.

An important application of control decoupling theory to the problem of aircraft dynamics was given by S. Singh and A. Schy, [27]. These authors derive a feedback for a simplified aircraft model in which certain aerodynamic forces had been ignored. One of the aims of this report is to produce a control decoupling feedback for a full blown model - the so-called aircraft H model. In Section 3.1 we will give what we believe to be the simplest derivation of a decoupling feedback control law together with a simple geometrical explanation of the method.

3. A Simpler 3rd Order Model

This section consists of three parts. In the first, we investigate the underlying geometry of control decoupling feedback, with the aim of gaining intuition about the problem of interest - the control of certain model of aircraft dynamics, the so called "aircraft H" model.

Second, we give a brief outline of Poincare's theory of normal forms, which is used to study the Hopf bifurcation. We follow the work of Poincare, Hopf, Andronov et al. and Arnold.

Third, we apply the above two ideas to study a model with three degrees of freedom, to illustrate the effect of control decoupling upon the Hopf bifurcation.

Using the theory of normal forms our goal is to reduce our system to a simpler form; the subsequent application of control decoupling feedback will illustrate the stabilizing (or destabilizing) effect of control decoupling upon the Hopf bifurcation. We shall also derive a criterion for determination of stability of the decoupled systems.

3.1 Derivation and Geometry of a Control Decoupling Feedback

Here we provide a straight forward derivation of a control decoupling feedback for the system

$$\dot{x} = F(x) + G(x)u$$

$$y = Hx \tag{3.1.1}$$

As we have remarked in Section 2, the objective is to modify the dynamics using state feedback in such a way that the i-th control input influences only the i-th output.

Differentiating the output and using the feedback $u=\alpha+\beta v$ (with $\alpha(x)$, $\beta(x)$ to be found) we obtain

$$\dot{y} = H\dot{x} = HF + HGu = (HF+HG\alpha) + HG\beta v$$

Our aim is to find α and β such that v_i controls y_i and only y_i. Such a choice is determined by setting $\dot{y}=\Lambda v$ (Λ is an arbitrary constant diagonal matrix), i.e. $HF+HG\alpha=0$, $HG\beta=\Lambda$.

We obtain:

$$\alpha = - (HG)^{-1}H F$$

$$\tag{3.1.2}$$

$$\beta = (HG)^{-1}\Lambda$$

the desired feedback is: $u=-(HG)^{-1}HF+(HG)^{-1}\Lambda v$.

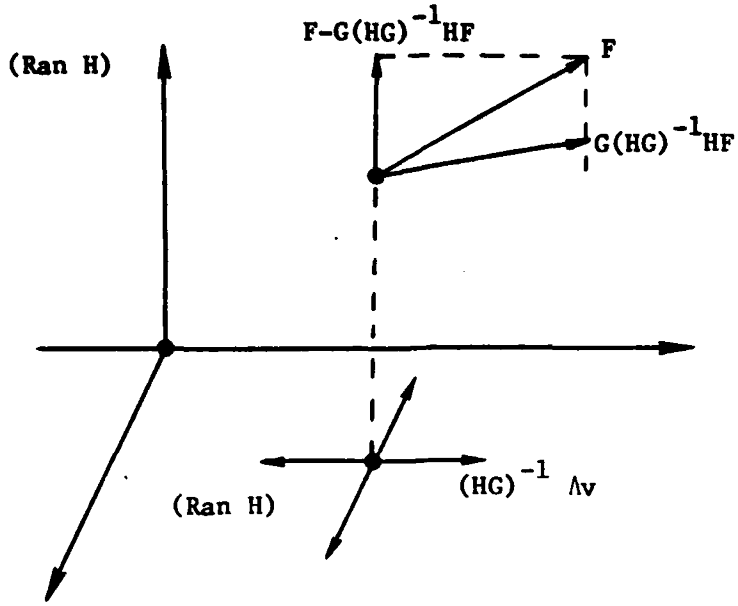


Figure This figure shows schematically the geometry of control decoupling feedback in dimensions 3 and higher

We briefly indicate the geometrical meaning of the terms in (3.1.2). The term $-(HG)^{-1}HF$ in the feedback law has the effect of annihilating the "horizontal" component of the vector field F, i.e. the component along Ran H.

The term "horizontal" is suggested by the aircraft H model, where H is the orthogonal projection onto a 3-dimensional subspace spanned by coordinate vectors. The effect of the part $(HG)^{-1}\Lambda$ in the feedback law is to align directions of input with directions of output, i.e. to take a vector $u=e_i$ parallel to the i-th axis in U-space into a vector $HG(HG)^{-1}\Lambda u=\Lambda e_i$ in the same direction.

3.2 Poincare's Theory of Normal Forms

One of the most powerful tools in studying dynamical system is the method of normal forms, which Poincare introduced in his dissertation. This method will be briefly described, and we refer the interested reader to Arnold (1983) for further details. The development of normal forms for nonlinear control systems can provide considerable insight into problems of synthesizing stabilizing feedback control laws. In general, by a "normal form" for a control system we mean a canonical form with respect to local diffeomorphic changes of state coordinates near an equilibrium for the force-free "drift" term. In terms of our general model, this drift term is given by the vector field F(x). The main idea behind Poincare's theory of normal forms is to reduce a system

$$\dot{x} = Ax + \text{higher order terms} \quad (x \in R) \qquad (3.2.1)$$

to as simple a form as possible by means of transformations $x=T(z)= z + \text{h.o.t.}$ Ideally, one would like to linearize (3.2.1), but the existence of such a linearizing transformation depends upon certain non-resonance conditions. That is, there may be Diophantine relations between the eigenvalues of A which prevent one from eliminating some higher order terms.

More precisely, a collection of eigenvalues $\mu=(\mu_1,\ldots,\mu_n)$ of the nxn matrix A is called <u>resonant</u>, if there is a relationship of the form $\mu_s=(m,\mu)$, where $m=(m_1,\ldots,m_n)$ with $m_k \geq 0$, $\Sigma\, m_k \geq 2$. Here $(.,.)$ denotes the usual dot product. Such a relationship is called a

resonance. For example, $\mu_1 + \mu_2 = 0$ is a resonance, or rather, it implies the resonance $\mu_1 = 2\mu_1 + \mu_2$, and more generally, $\mu_1 = (k+1)\mu_1 + k\mu_2$, $\mu_2 = k\mu_1 + (k+1)\mu_2$, $k = 1,2,\ldots$. Let $x_1, \ldots, x_n$ be the coordinates in the eigenbasis $e_1, \ldots, e_n$ of the matrix A. Equivalently, we can perform a preliminary linear transformation diagonalizing A; then e_s will have coordinates

$$e_s = \text{col}\,(0,\ldots,1_s,\ldots,0), \text{ and } A = \text{diag}\,(\mu_1,\ldots,\mu_n).$$

Therefore, there is no loss of generality in assuming that A is diagonal (we omit the case of Jordan blocks, which can be treated without difficulty, too).

Consider a vector monomial $x^m e_s$, where $x^m = x_1^{m_1} \ldots x_n^{m_n}$ is a scalar monomial, and e_s is an eigenvector of A. It is called resonant if $\mu_s = (m,\mu)$, $m \geq 2$.

Now, the main theorem (of Poincare and Dulac) asserts that the system

$$\dot{x} = Ax + \ldots,$$

can be reduced by a (formal) change of variables $x = y + \ldots$ to the form

$$\dot{y} = Ay + w(y)$$

where all the vector monomials in the series w are resonant. We give a short proof of this theorem in Appendix 1.

We illustrate Poincare-Dulac's theorem by applying it to the proof of the existence of a Hopf bifurcation.

Consider the systems (cf. Section 3.3)

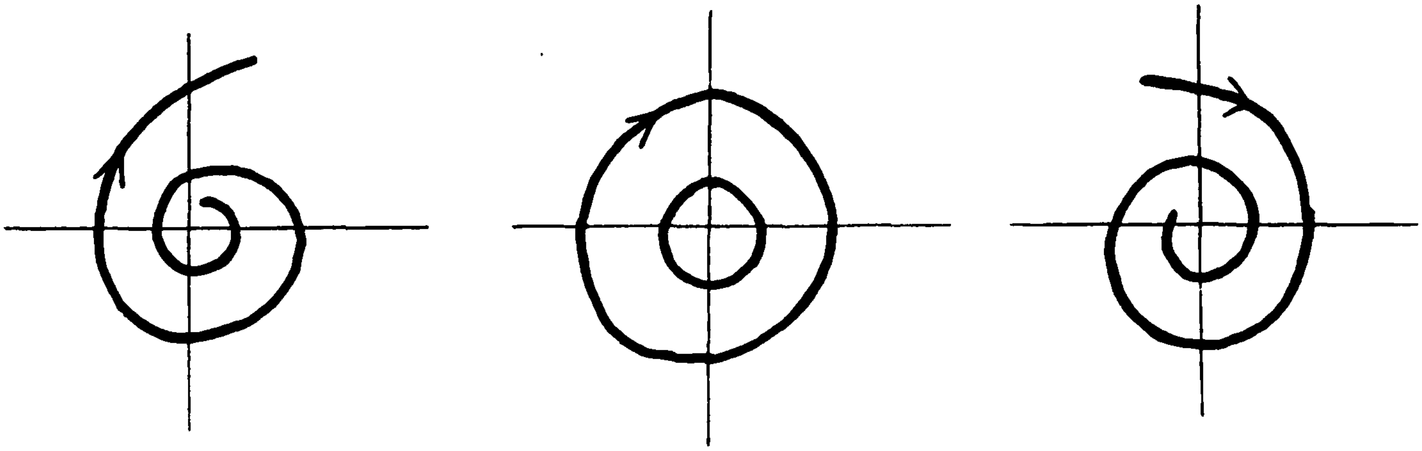
$$\begin{aligned} \dot{x} &= \lambda x + \omega y + \ldots \\ \dot{y} &= -\omega x + \lambda y + \ldots \end{aligned} \tag{3.2.2}$$

where λ is a parameter near 0. When $\lambda=0$, we have a resonance, $\mu_1 + \mu_2 = 0$ ($\mu_{1,2} = \pm i\omega$), and therefore for $\lambda=0$ we cannot kill the resonant terms of third, fifth, etc. order. Therefore, we do not try to kill them at all, even for $\lambda=0$ - an attempt to do so would lead to the discontinuous dependence of the vector field on the parameter λ. Eliminating the second and fourth order terms by a change of variables, we can reduce (3.2.2) to the form

$$\begin{aligned} \dot{x}_1 &= \lambda x_1 + \omega y_1 + (\alpha x_1 + \beta x_2)(x_1^2 + y_1^2) + 0_5 \\ \dot{y}_1 &= -\omega x_1 + \lambda y_1 + (-\beta x_1 + \alpha x_2)(x_1^2 + y_1^2) + 0_5 \end{aligned} \tag{3.2.3}$$

This reduction also justifies our seemingly specific choice of the example in Section 3.3.

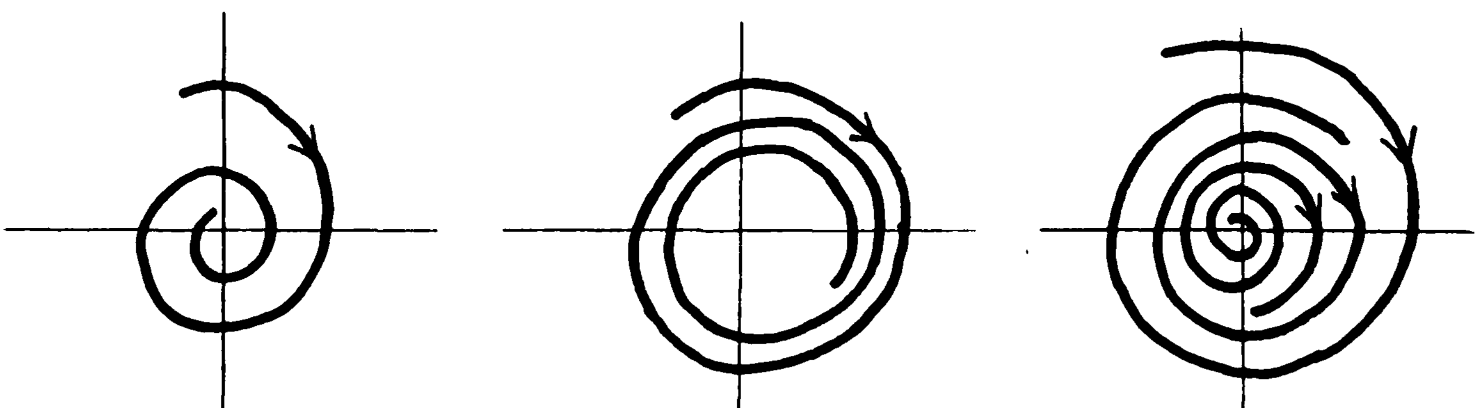
To eludicate the geometrical meaning of this analysis, we refer to Figure 3.2-1.



$\lambda>0$ $\lambda=0$ $\lambda<0$

(Degenerate) bifurcation (at $\lambda=0$) of the linear system

$$\left.\begin{aligned}\dot{x} &= \lambda x + \omega y\\ \dot{y} &= -\omega x + \lambda y\end{aligned}\right., \quad \omega>0$$



0 exponentially stable
$\lambda<0$

0 is stable, non-exponentially
$\lambda=0$

0 is exponentially unstable, periodic orbit has bifurcated off 0.

$\lambda>0$

Hopf Bifurcation (at $\lambda=0$) of the nonlinear system

$$\left.\begin{aligned}\dot{x} &= \lambda x + \omega y + (\alpha x + \beta y)\,(x^2+y^2)\\ \dot{y} &= -\omega x + \lambda y + (-\beta x + \alpha y)\,(x^2+y^2)\end{aligned}\right., \quad \lambda<0$$

Figure 3.2-1

Introducing the distance $r = (x_1^2 + y_1^2)$ from the origin, we obtain the the following ODE for r:

$$\dot{r} = \lambda r + \alpha r^2 + 0_3,$$

where 0_3 refers to the terms of order r^3 and higher.

Ignoring 0_3, we obtain the equation

$$r = (\lambda + \alpha r)r \quad (r > 0)$$

for which $r_0 = 0$ is always a stationary point (which corresponds to the equilibrium solution of (3.2.2)); another stationary point $r_1 = -\lambda/\alpha$, (if $\frac{\lambda}{\alpha} < 0$) corresponds to a periodic solution of the truncated system (3.2.3)). Actually, the presence of 0_5 in (3.2.3) will not destroy this solution but will only perturb it slightly, as can be easily shown. This periodic solution is close to a circle of radius $\sqrt{-\lambda/\alpha}$; it bifurcates off the origin as λ crosses 0. This is exactly the Hopf bifurcation.

During almost a century of its existence, Poincare's method has been used by many researchers, including Birkhoff, Hopf, Moser and others, in conjunction with their work on problems arising in such applications as electrical engineering and classical and celestial mechanics, to name but a few.

3.3 An Illustrative Example of Hopf Bifurcation in a Control System

It is known that the aircraft H model of section 4 enters an oscillatory regime as angle of attack becomes large. In Mehra, Kessel and Carroll (1978) it was demonstrated that this instability is due to Hopf bifurcation. Our eventual goal is to determine the possibility of stabilizing this system via decoupling feedback. (We construct such a feedback in section 4, by applying the method given in section 3.1 above).

Before attempting this derivation, we would like to understand the geometry of the problem; to that end, instead of the aircraft model, as a first step we consider a simpler three-dimensional system with two controls, which *retains the essential* instability of the aircraft model, but is easier to visualize geometrically.

The role of equilibrium angle of attack (or equivalently elevator angle δe) will be played by a certain bifurcation parameter λ in our example. In analogy with the angle of attack in the aircraft model, as λ increases, our system will pass from stable steady state to an oscillatory regime.

We will then introduce a control decoupling feedback into our system and will determine its effect upon stability. The importance of considering this example is that is suggests the direction of analysis for the actual aircraft model.

Thus, we consider a family of systems

$$\dot{x} = F(x,\lambda,\alpha) + G(x)u, \tag{3.3.1}$$

$$y = Hx \tag{3.3.2}$$

with $x \in R^2$, $u \in R^2$ and $H = \begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$ being the matrix of projection of the vector (x_1,x_2,x_3) on the (x_1,x_2)-plane.

The parameter λ will be roughly analogous to the equilibrium angle-of-attack, while α will characterize the geometry of the system and will be referred to later.

One of our major techniques for analyzing stabilization schemes for (3.3.1)-(3.3.2) is to put this control system into a <u>normal form</u>, by a change of coordinates in the state variables which is locally defined in a neighborhood of an equilibrium point. Note that such a change of coordinates does not change the time trajectories (y(t), u(t)) of (3.3.1)-(3.3.2), but rather enables us to define a feedback strategy for (3.3.1) in the context of the simpler, normal form.

We first describe the free motion of (3.3.1) in a geometric fashion. To be specific, consider the third order control system

$$\begin{aligned} \dot{x}_1 &= \lambda x_1 + \omega c x_2 + \omega s x_3 - x_1(x_1^2 + (cx_2 + sx_3)^2) \\ &\quad + g_{11}u_1 + g_{12}u_2 \\ \dot{x}_2 &= -\omega c x_1 + (\lambda c^2 - s^2)x_2 + (\lambda+1)csx_3 - c(cx_2 + sx_3)(x_1^2+(cx_2+sx_3)^2) \\ &\quad + g_{21}u_1 + g_{22}u_2 \end{aligned} \tag{3.3.3}$$

$$\dot{x}_3 = -\omega s x_1 + (\lambda+1)cs x_2 + (\lambda s^2 - c^2)\, x_3 - s(cx_2 + sx_3)(x_1^2+(cx_2+sx_3)^2) + g_{31}u_1 + g_{32}u_2$$

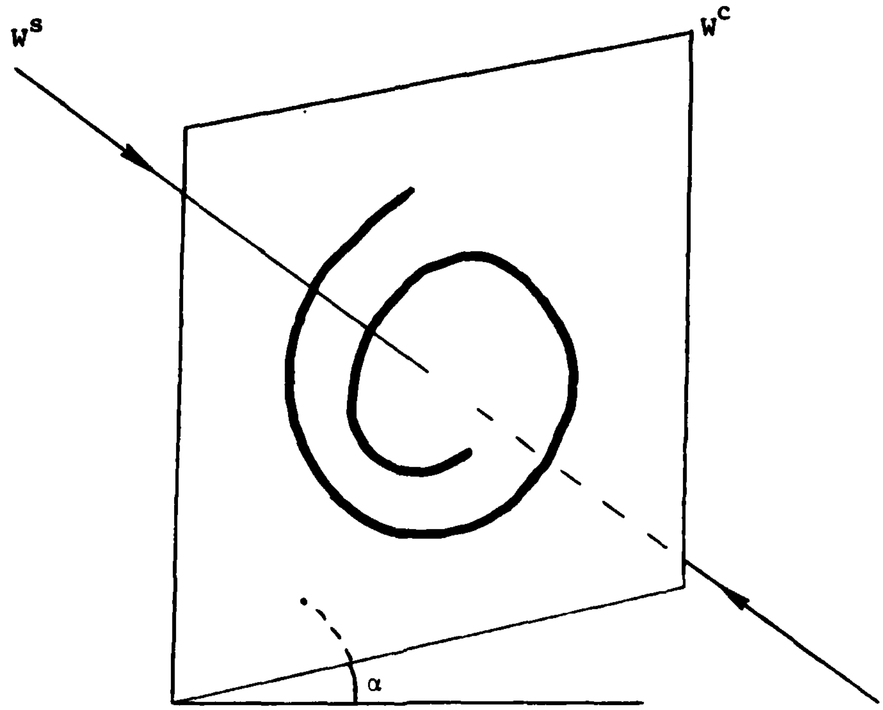
where $G(x)=(g_{ij}(x))$ is assumed to be of full rank in a neighborhood of 0, ω is a fixed real quantity, $c=\cos\alpha$, $s=\sin\alpha$, and λ and α are parameters. Consider the free motion of this system (with controls $u_1=u_2=0$)

$$\begin{aligned}
\dot{x}_1 &= \lambda x_1 + \omega c x_2 + \omega s x_3 - x_1(x_1^2 + (cx_2 + sx_3)^2) \\
\dot{x}_2 &= -\omega c x_1 + (\lambda c^2 - s^2)\, x_2 + (\lambda+1)cs x_3 - c(cx_2 + sx_3)(x_1^2+(cx_2+sx_3)^2) \\
\dot{x}_3 &= -\omega s x_1 + (\lambda+1)cs x_2 + (\lambda s^2 - c^2)\, x_3 - s(cx_2 + sx_3)(x_1^2+(cx_2+sx_3)^2)
\end{aligned} \tag{3.3.4}$$

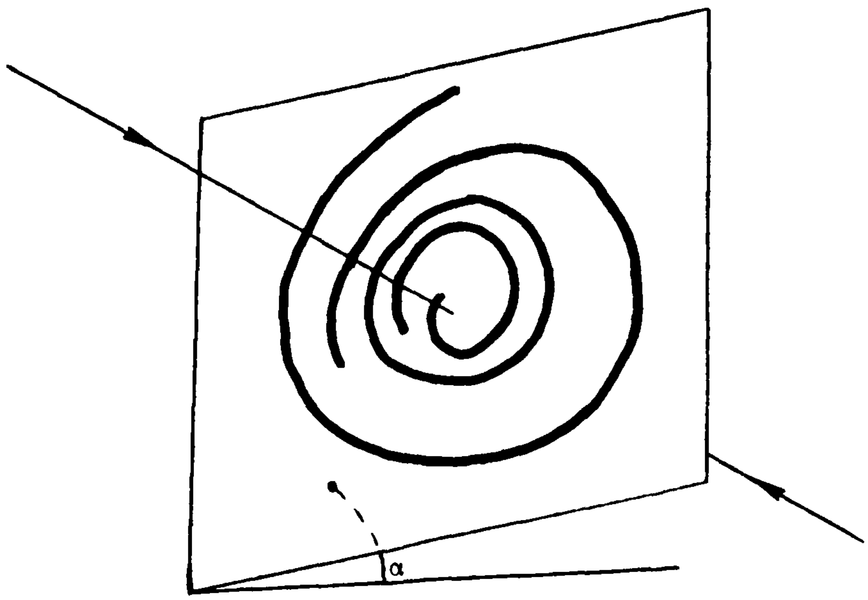
Standard results (see, for example Marsden and McCracken, [ ]) show that as λ (a real parameter) increases from negative values to positive ones, a limit cycle for (3.3.4) appears. (See figure 3.3-1) Indeed, if we fix the parameter α, this system represents a normal form for third order families of systems admitting a Hopf bifurcation The limit cycle is confined to an invariant two dimensional manifold W^c (which for $\lambda=0$ is called the center manifold). The other invariant manifold, W^s, is stable for all λ, and it intersects W^c transverally. Note that there is a plane through the origin perpendicular to W^s, and the motion of the system restricted to this plane is given by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \lambda x + \omega y - x(x^2+y^2) \\ -\omega x + \lambda y - y(x^2+y^2) \end{pmatrix},$$ where x,y are coordinates in that plane.

We point out that, although this picture may seem to be very specific,



$\lambda < 0$



$\lambda > 0$

Figure 3.3-1

it accurately represents what actually occurs in the aircraft model, except that in the latter $\dim W^s=5$ instead of 1 and $\dim (W^s)^\perp=3$ instead of 2.

According to section 3.1, the feedback

$$u = -(HG)^{-1}H\,F(x,\lambda,\alpha) + (H\,G)^{-1}\Lambda v$$

decouples the closed loop system. The resulting system is

$$\dot{x}_1 = \lambda_1 v$$
$$\dot{x}_2 = \lambda_2 v$$
$$x_3 = F_3 - \beta_1 F - \beta_2 F_2 + \beta_1\lambda_1 v_1 + \beta_2\beta_2 v_2,$$

where $G(H\,G)^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \beta_1 & \beta_2 \end{pmatrix}$, $F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$ and β_1, β_2 are functions of x.

More explicitly, we have

$$\dot{x}_1 = \lambda_1 v_1$$
$$\dot{x}_2 = \lambda_2 v_2$$
$$x_3 = Ax_1 + Bx_2 + Cx_3 + N(x_1, x_2, x_3) + \beta_1\lambda_1 v_1 + \beta_2\lambda_2 v_2,$$

where

$$A = -\omega s - \beta_1\lambda + \beta_2\omega c$$
$$B = (\lambda H)cs - \beta_1\omega c - \beta_2(\lambda c^2 - s^2)$$
$$C = (\lambda s^2 - c^2) - \beta_1\omega s - \beta_2(\lambda+1)cs,$$

and N includes cubic terms.

Now, to determine stability of this system, we keep $x_1=x_2=0$ (by setting $v_1=v_2=0$ and making initial values of x_1, x_2 vanish); the resulting system for x_3 is $\dot{x}_3=C(\lambda,\alpha)x_3+N(0,0,x_3)$; it is stable precisely when $C(\lambda,\alpha)<0$.

Thus, we obtain a <u>stability criterion:</u> <u>If $C(0,\alpha)<0$, then Hopf bifurcation is "stabilized" by the control feedback (3), i.e. as long as x_1, x_2 remain small, so does x_3.</u>

Explicitly, this criterion is:

$$-c^2 - \beta_1 \omega s - \beta_2 \omega c s < 0$$

The importance of this criterion is that it points to the fact that either situation can occur: the feedback may either stabilize or destablilize the bifurcation. For instance if we have $\beta_1=\beta_2=\omega=1$, the criterion becomes

$$\cos^2\alpha + \sin\alpha + \cos\alpha \sin\alpha > 0;$$

for some angles α(e.g. $\alpha=0$) this inequality holds, and for others (e.g. $\alpha=-\frac{\pi}{4}$) it doesn't. In each specific case one would have to make calculations to determine the sign of C.

When this approach is generalized to the aircraft model, we obtain a similar criterion; instead of the sign of the coefficient C, we have to check stability of a certain 4x4 matrix (corresponding to the uncontrolled variables). The details of this are sketched in

section 4 where the state dynamics for x_1, x_4 and x_5 is decoupled, but the state dynamics for x_2, x_3, x_6 and x_7 is further complicated. Flight experience with advanced fighters such as F-14 shows that certain control designs e.g. aileron-rudder interconnect, can produce problems in other flight regimes. We believe further research along the lines we have indicated will lead to analytical methods for solving such problems by examining the global stability properties of given feedback laws.

4. Application to the Aircraft H Model

In this section we apply the technique described in section 3-1 above to the problem of control decoupling in the full aircraft H model and give the method to determine stability of the resulting decoupled system.

The seven degree of freedom aircraft model (Hecker and Oprisiu, 1976) has the form:

$$\dot{p} = \ell_\beta \beta + \ell_{\alpha\delta a} \Delta\alpha\delta a + \ell_q q + \ell_r r + \ell_{\beta\alpha} \beta\Delta\alpha + \ell_{r\alpha} r\Delta\alpha + \ell_p p - i_1 qr + \ell_{\delta a} \delta a + \ell_{\delta r} \delta r$$

$$\dot{q} = \bar{m}_\alpha \Delta\alpha + \bar{m}_q q + i_2 pr + m_{\delta e}\, \delta e - m_{\dot{\alpha}} p\beta$$

$$\dot{r} = n_\beta \beta + n_{\alpha\delta a}\, \Delta\alpha\delta a + n_r r + n_p p + n_{p\alpha} p\Delta\alpha - i_3 pq + n_{\delta\alpha}\, \delta a + n_{\delta r}\, \delta r$$

$$\dot{\alpha} \simeq q - p\beta + z_\alpha \Delta\alpha + z_{\delta e}\, \delta e + (q/V)\,(\cos\theta \cos\phi - \cos\theta_o) \qquad (4.1)$$

$$\dot{\beta} = y_\beta \beta + p(\sin(\alpha + \Delta\alpha) - r \cos\alpha_o + y_{\delta a}\, \delta a + y_{\delta r}\, \delta r + (g/V) \cos\theta \sin\phi$$

$$\dot{\phi} = p + q \tan\theta \sin\phi + r \tan\theta \cos\phi$$

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

where $\bar{m}_\alpha = m_\alpha + m_{\dot{\alpha}} z_\alpha$, $\bar{m}_q = m_q + \bar{m}_{\dot{\alpha}}$, $m_{\delta e} = m_{\delta e} + m_{\dot{\alpha}} z_{\delta e}$, and where the symbols are as follows:

x,y,z principal axes of aircraft

I_x, I_y, I_z moments of inertia about principal axes (kg/m^2)

$$\left.\begin{array}{l} i_1 = (i_z - I_y)/I_x \\ i_2 = (I_z - I_x)/I_y \\ i_3 = (I_y - I_x)/I_z \end{array}\right\} \text{ non-dimensional inertia coefficients}$$

p,q,r Angular velocity components along roll, pitch and yaw axes (principal axes) of aircraft (rad/sec)

V velocity of aircraft's center of mass (m/sec)

g gravitational acceleration (m/sec^2)

α angle of attack (rad)

β angle of sideslip (rad)

θ pitch angle (rad)

ϕ angle of bank (rad)

δa, δr, δe aileron, rudder, and elevator deflection angles (rad)

rolling moment per I_x ($1/sec^2$)

m pitching moment per I_y ($1/sec^2$)

n yawing moment per I_z ($1/sec^2$)

y side force over (aircraft mass times speed) (1/sec)

z aerodynamic force along principal axis z over mass times speed (1/sec)

$\alpha, \beta, \dot{\alpha}, p, q, r, \delta a, \delta r, \delta e$ as subscripts denote partial derivatives with respect to the respective quantity, i.e.

$$y_\beta = \frac{\partial y}{\partial \beta}, \quad \ell_{\delta a} = \frac{\partial \ell}{\partial \delta a}, \quad m_{\dot{\alpha}} = \frac{\partial m}{\partial \frac{d\alpha}{dt}}, \quad n_{\delta a \alpha} = \frac{\partial^2 n}{\partial \alpha \partial \delta a}$$

ϕ_c bank angle command

We change the notation to a more uniform one by setting $(p,q,r,\alpha,\beta,\phi,\theta) = (x_1,x_2,x_3,x_4,x_5,x_6,x_7)$, $\ell_p = \ell_1, \ell_g = \ell_2$, etc. (the index of each coefficient corresponds to the index of the variable (or variables) which it multiplies).

Our system can be rewritten as

$$\dot{x} = F(x) + G(x)u \ , \ y = Hx \tag{4.2}$$

where

$$x = \begin{pmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_7 \end{pmatrix} \ , \quad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \ , \quad G = \begin{bmatrix} a_1 & 0 & a_2 \\ 0 & a_3 & 0 \\ a_4 & 0 & a_5 \\ 0 & a_6 & 0 \\ a_7 & 0 & a_8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \end{bmatrix} ;$$

the explicit form of $F_i(x)$ is evident from (4.1), and a_i are constants, except for a_1 and a_4:

$$\begin{aligned} a_1 &= \ell_{\alpha\delta a}\Delta x_4 + \ell_{\delta z}, & a_2 &= \ell_{\delta r} \\ a_3 &= m_{\delta e} & a_4 &= n_{\alpha\delta a}\Delta x_4 + n_{\delta\alpha} \\ a_5 &= n_{\delta r} & a_6 &= z_{\delta e} \\ a_7 &= y_{\delta a} & a_8 &= y_{\delta r} \end{aligned}$$

The procedure described in section 3-1 leads to the following results (we omit the tedious calculations).

The functions $\alpha(x)$ and $\beta(x)$ in the control decoupling feedback $u = \alpha(x)+\beta(x)v$ are as follows:

$$\beta(x) = (HG)^{-1}\Lambda = \Delta^{-1}\begin{pmatrix} h_1 & 0 & h_2 \\ 0 & h_3 & 0 \\ h_4 & 0 & h_5 \end{pmatrix}\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}\begin{pmatrix} \lambda_1 h_1 & 0 & \lambda_3 h_2 \\ 0 & \lambda_1 h_3 & 0 \\ \lambda_1 h_4 & 0 & \lambda_3 h_5 \end{pmatrix} \tag{4.3}$$

where $\Delta = a_1a_6a_8 - a_2a_6a_7$ and $h_1 = a_6a_8$, $h_2 = -a_2a_6$, $h_3 = -a_2a_7 + a_1a_8$, $h_4 = -a_6a_7$, $h_5 = a_1a_6$

$$\alpha(x) = -(HG)^{-1}H\,F = \Delta^{-1}\begin{pmatrix} F_1h_1 + F_5h_2 \\ F_4h_3 \\ F_1h_4 + F_5h_5 \end{pmatrix} \tag{4.4}$$

The feedback $u = \alpha(x)+\beta(x)v$ results in the following system:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ F_1 - A_1F_4 \\ F_3 - A_2F_1 - A_3F_5 \\ 0 \\ 0 \\ F_6 \\ F_7 \end{bmatrix} + \begin{bmatrix} \lambda_1 v_1 \\ A_1\lambda_2 v_2 \\ A_2\lambda_1 v_1 + A_3\lambda_3 v_3 \\ \lambda_2 v_2 \\ \lambda_3 v_3 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 = \Delta^{-1}(a_{138} - a_{237})$$

where
$$A_2 = \Delta^{-1}(a_{468} - a_{567})$$
$$A_3 = \Delta^{-1}(a_{156} - a_{246})$$

with a_{138} standing for $a_1a_3a_8$, etc. Local stability of the uncontrolled part of the system depends on the eigenvalues of the matrix of the linearization of the dynamical system restricted to the 4-dimensional subspace given by $x_1=x_5=0$, $x_4=x_4^0 \sim 2.5°$ (the leveled steady flight angle of attack). Explicitly, this dynamical system has the following form:

$$\begin{cases} \dot{x}_2 = F_1 - A_1F_4 & + \quad A_1 \lambda_2 v_2 \\ \dot{x}_3 = F_3 - A_2F_1 - A_3F_5 & + \quad A_2\lambda_1 v_1 + A_3\lambda_3 v_3 \\ \dot{x}_6 = F_6 & \\ \dot{x}_7 = F_7 & \end{cases}$$

Here the components F_i of the vectorfield are as follows:

$$F_1 = \ell_5x_5 + \ell_2x_2 + \ell_3 x_3 + \ell_{45}\Delta x_4x_5 - \ell_{34}x_3\Delta x_4 + \ell_1x_1 - i_1x_2x_3$$
$$F_3 = n_5x_5 + n_3x_3 + n_1x_1 + n_{14}x_1\Delta x_4 - i_3x_1x_2$$
$$F_4 = x_2 - x_1x_5 + z_4 \Delta x_4 + \frac{g}{V} (\cos x_7 \cos x_6 - \cos x_7^\circ)$$
$$F_5 = y_5x_5 + x_1 \sin x_4 - x_3\cos x_4^\circ + \frac{g}{V} \cos x_7 \sin x_6$$
$$F_6 = x_1 + x_2 \tan x_7 \sin x_6 + x_3 \tan x_7 \cos x_6$$
$$F_7 = x_2 \cos x_6 - x_3 \sin x_6$$

The resulting nonlinear system becomes, after setting $x_1=x_5=0$, $x_4 = x_4^\circ$ (so that $\Delta x_4=0$):

$$\dot{x}_2 = (\ell_2 - A_1)\, x_2 + \ell_3x_3 - i_1x_2x_3 \frac{A_1 g}{V} \cos x_6 \cos x_7 + A_1 \frac{g}{V} \cos x_7^\circ$$

$$\dot{x}_3 = -\ell_2 A_2 x_2 + (n_3 - A_2 \ell_3 + A_3 \cos x_4^\circ)\, x_3 + A_2 1_1 x_2 x_3 - \frac{A_3 g}{V} \cos x_7 \sin x_6 .$$

$$\dot{x} = x_2 \tan x_7 \sin x_6 - x_3 \tan x_7 \cos x_6$$

$$\dot{x}_7 = x_2 \cos x_6 - x_3 \sin x_6 .$$

To determine stability of the equilibrium position $x_2 = x_3 = x_6 = 0$, $x_7 = x_7^\circ \sim 2.5°$, we linearize the system near this point.

The resulting linear system is

$$\dot{\xi}_2 = (\ell_2 - A_1)\xi_2 + \ell_3 \xi_3 + \frac{A_1 g}{V} \sin x_7^\circ\, \xi_7$$

$$\dot{\xi}_3 = -\ell_2 A_2 \xi_2 + (n_3 - A_2 \ell_3 + A_3 \cos x_4^\circ)\xi_3 - \frac{A_3 g}{V} \cos x_7^\circ\, \xi_6$$

$$\dot{\xi}_6 = -\tan x_7^\circ\, \xi_3$$

$$\dot{\xi}_7 = \xi_2$$

Our aim is to determine the eigenvalues of this system. After a tedious calculation, we can rewrite the system as follows:

$$\dot{\xi}_2 = -174.9\, \xi_2 + .126\, \xi_3 + .276\, \xi_7$$

$$\dot{\xi}_3 = -3.38\, \xi_2 - 5438\, \xi_2 + 201\, \xi_6$$

$$\dot{\xi}_6 = -.0436\, \xi_3$$

$$\dot{\xi}_7 = \xi_2$$

The eigenvalues of the matrix of this system are computed to be:

$\lambda_1 = -5438$, $\lambda_2 = -174.9$, $\lambda_3 = -1.6$, $\lambda_4 = 0.0016$;

thus the system has a very weak instability, which in practice should amount to a neutral equilibrium.

Our analysis shows, therefore, that the decoupling of controls in the specific case under consideration does not alter weak instability present in a horizontal steady flight in any significant way.

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Appendix I

Here we prove the theorem of Poincare and Dulac which is restated here for the reader's convenience.

<u>Theorem.</u> The system

$$\dot{x} = Ax + v(x), \quad A = \mathrm{diag}(\mu_1, \ldots, \mu_n) \tag{A1}$$

can be reduced by a formal change of variables $x=y+\ldots$ to the form

$$\dot{y} = Ay + w(y)$$

where all the vector monomials $y^m \ell_s$ comprising the series $w(y)$ are resonant (for definitions see section 3.2).

<u>Proof.</u> Performing the change of variables $x=y + h(y)$ in (A1) (here lowest degree in $h(y)$ is ≥ 2), we obtain:

$$\dot{y} = Ay + Ah(y) - h'(y)\,Ay + v(y) + \text{h.o.t.} \tag{A2}$$

Let $k\geq 2$ be the lowest degree of the vector monomials $y^m \ell_s$ comprising the series $v(y)$; in order to eliminate these terms we must solve for h the equation

$$h'(y)\,Ay-Ah(y) = v_k(y), \tag{A3}$$

where $v_k(y)$ retains only the monomials from $v(y)$ of degree k.

Our aim is, therefore, to study the properties of the operator

$$(Lh)(y) = h'(y)Ay - Ah(y)$$

mapping the set of polynomial vectorfields $h(y)$ of degree k into itself. The complete information about L is contained in the following

Lemma. The eigenvalues of L are the vectorfields of the form $y^m \ell_s$; the corresponding eigenvalues are $(\mu,m)-\mu_s$.

Postponing for a moment the proof of the lemma, we finish the proof of the theorem. Using the lemma, we can eliminate all nonresonant monomials of degree k in the right hand side of (A3), by the proper choice of the terms of degree k in $h(y)$. Similarly, we can kill all nonresonant terms of degree $(k+1)$ without affecting any terms of lower degree - indeed, the transformation $x=y+h_{k+1}(y)$ does not affect terms of degree $\leq k$, as follows from (A2).

Inductive application of this argument proves the theorem.

Proof of the lemma.

$$L(y^m \ell_s) = (y^m \ell_s)^1 \; Ay - A(y^m \; \ell_s) =$$

$$= \begin{bmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ \frac{m_1}{y_1} y^m & \cdot & \cdot & \cdot & \frac{m_n}{y_n} y^m \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} \mu_1 y_1 \\ \vdots \\ \mu_n y_n \end{bmatrix} - y^m \mu_s \ell_s =$$

$$= \begin{bmatrix} 0 \\ \cdot \\ (m,\mu)y^m \\ \cdot \\ \cdot \\ 0 \end{bmatrix} - y^m \begin{bmatrix} 0 \\ \cdot \\ \mu_s \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = [(m,\mu)-\mu_s]\,(y^m \ell_s\ ),$$

q.e.d.

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